Available online at www.sciencedirect.com

SCIENCE DIRECT®

International Journal of Heat and Fluid Flow 24 (2003) 23–28

International Journal of HEAT AND FLUID FLOW

www.elsevier.com/locate/ijhff

Transient heat transfer measurements using thermochromic liquid crystal. Part 2: Experimental uncertainties

J. Michael Owen *, Paul J. Newton, Gary D. Lock

Department of Mechanical Engineering, University of Bath, Bath BA2 7AY, UK

Received 28 January 2002; accepted 4 August 2002

Abstract

In Part 1 of this two-part paper, an "exponential-series technique" was used to calculate heat transfer coefficient, h, for the so-called slow transient case where it is not possible to generate a step-change in the air temperature. Small uncertainties in the measured temperatures can, however, create large uncertainties in the calculated value of h, and the amplification parameter, Φ_h, is defined as the ratio of the relative uncertainty in h to the relative uncertainties in the temperatures. Using an uncertainty analysis, theoretical expressions for Φ_h are found for the slow transient case, and these expressions are in excellent agreement with values computed using a Monte Carlo method. The results provide guidance in the selection of design parameters for an experiment and for the calculation and minimisation of the uncertainty in h.

Keywords: Uncertainties; Transient heat transfer; Thermochromic liquid crystal

1. Introduction

Part 1 of this two-part paper (see Newton et al., 2003, which is referred to below as 1) describes the "exponential-series technique". This technique can be used to determine the heat transfer coefficient, h, for "slow-transient" cases where it is not possible to generate a step-change in the air temperature.

Yan and Owen (2002) used the results of Coleman and Steele (1999) to provide an uncertainty analysis for heat transfer coefficients derived from the step-change solutions of Fourier's equation. If the random uncertainties in the measured temperatures are known then the uncertainty in h can be readily determined. This enables the transition temperature of the thermochromic liquid crystal (TLC) to be chosen so as to minimise the uncertainty in h.

It should be pointed out that, like the exponential-series technique described in 1, the uncertainty analysis discussed below assumes that h is invariant with time. Whilst this may not always be true in practice, the sensitivity of h to the (time-dependent) level or distribution of surface temperature can be tested by using two or more narrow-band liquid crystals to determine h.

It should also be pointed out that the uncertainty analysis assumes random uncertainties in the measured temperatures. Systematic uncertainties or biases (due, for example, to the calibration of the liquid crystal using the same thermocouple employed in the calibration of the air temperature) are beyond the scope of this paper.

The step-change analysis of Yan and Owen is extended below to the slow transient case. The uncertainties in h are quantified in Section 2, and Section 3 describes how to calculate the amplification parameters, which relate the uncertainties in h to those in the measured temperatures. The conclusions are summarised in Section 4. Using these results, experimenters should be able to quantify and to minimise the uncertainties in h resulting from the use of narrow-band TLC in slow-transient heat-transfer experiments.

2. Experimental uncertainties

The symbols used throughout this paper are defined in the Nomenclature of 1.

* Corresponding author. Tel.: +44-01225-386-115; fax: +44-01225-386-928.
E-mail address: j.m.owen@bath.ac.uk (J.M. Owen).

0142-727X/02/$ - see front matter © 2002 Elsevier Science Inc. All rights reserved.
PII: S0142-727X(02)00207-2

2.1. Solution of Fourier's equation for the "exponential-series case"

For convenience, Eq. (2.22) of 1 is rewritten below as

$$\Theta = \frac{T_w - T_0}{T_{aw,\infty} - T_0} = \sum_{j=1}^{m} c_j g(\beta, \lambda_j), \tag{2.1}$$

where

$$\lambda_j = \frac{\beta}{\beta_{\tau,j}} = \sqrt{\frac{t}{\tau_j}}, \tag{2.2}$$

$$c_j = \frac{T_{a,j}}{\sum_{j=1}^{m} T_{a,j}} \tag{2.3}$$

and

$$g(\beta, \lambda_j) = 1 - \frac{\lambda_j^2}{\beta^2 + \lambda_j^2}[1 - f(\beta)] - \frac{\beta^2}{\beta^2 + \lambda_j^2} e^{-\lambda_j^2}\left[1 + \frac{\lambda_j}{\beta}\phi(\lambda_j)\right], \tag{2.4}$$

where $f(\beta)$ is defined in Eq. (2.6) of 1 and

$$\phi(\lambda_j) = \frac{1}{\pi}\left[\lambda_j + 2\sum_{n=1}^{\infty} \frac{e^{-n^2/4}}{n} \sinh(n\lambda_j)\right]. \tag{2.5}$$

Fig. 1 shows the effect of λ (for the case where $m = 1$ and $\lambda = \lambda_1$) on the variation of β with Θ according to Eq. (2.1). As $\lambda \to \infty$, $g(\beta, \lambda) \to f(\beta)$; that is, the step-change solution is the limit to the exponential solution. The two solutions are in close agreement for $\lambda > 2.2$.

As $\beta \to \infty$, Eq. (2.4) reduces to

$$g(\beta, \lambda_j) = 1 - e^{-\lambda_j^2}. \tag{2.6}$$

That is, as $h \to \infty$, the surface of the wall immediately reaches the instantaneous adiabatic-wall temperature.

2.2. Uncertainty in h

Consider the case where the random uncertainties in the measured values of T_w, T_0 and T_{aw} are independent of each other (as is often the case) then, using the method of Coleman and Steele (1999), it can be shown that

$$P_\beta^2 = P_\Theta^2 \Big/ \left(\frac{\partial\Theta}{\partial\beta}\right)^2, \tag{2.7}$$

where P_β and P_Θ are the uncertainties in β and Θ, respectively, and

$$P_\Theta^2 = \left(\frac{\partial\Theta}{\partial T_w}\right)^2 P_{T_w}^2 + \left(\frac{\partial\Theta}{\partial T_0}\right)^2 P_{T_0}^2 + \left(\frac{\partial\Theta}{\partial T_{aw}}\right)^2 P_{T_{aw}}^2, \tag{2.8}$$

where P_{T_w}, P_{T_0} and $P_{T_{aw}}$ are the respective uncertainties in the temperatures. It follows from the definition of Θ that

$$P_\Theta^2 = \left[\left(\frac{P_{T_w}}{\theta_{aw}}\right)^2 + (1-\Theta)^2\left(\frac{P_{T_0}}{\theta_{aw}}\right)^2 + \Theta^2\left(\frac{P_{T_{aw}}}{\theta_{aw}}\right)^2\right], \tag{2.9}$$

where

$$\theta_{aw} = T_{aw} - T_0. \tag{2.10}$$

It is assumed here that the relative uncertainties in t and κ are much smaller than those in h, so that

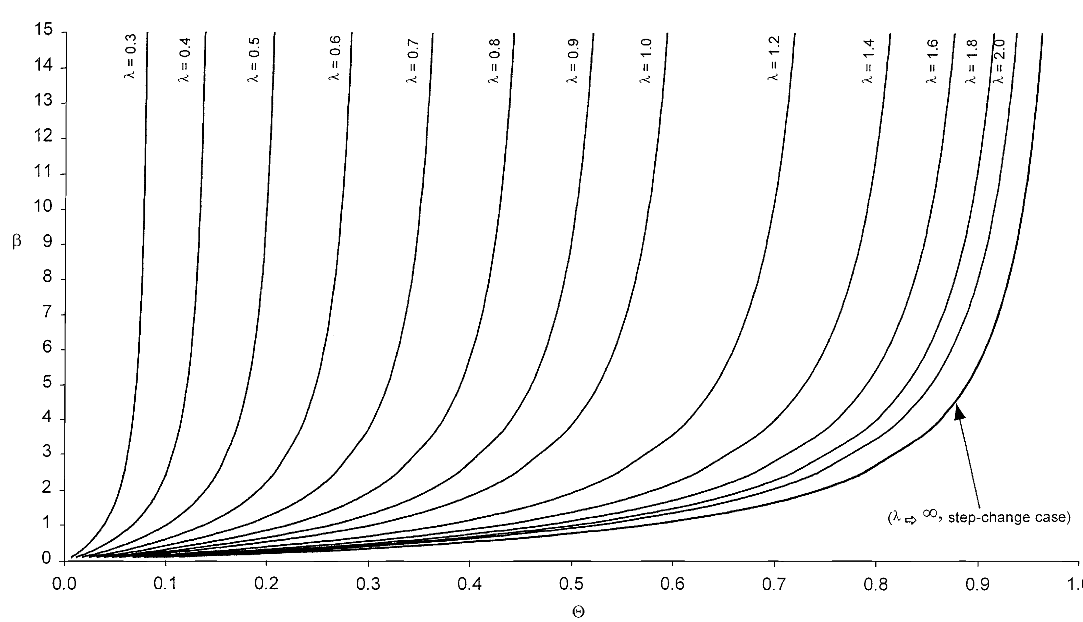

Fig. 1. Effect of λ on variation of β with Θ for exponential case.

$$\frac{P_\beta}{\beta} \approx \frac{P_h}{h}. \tag{2.11}$$

(Although it may not be insignificant, an uncertainty in κ will produce a *bias*, or systematic error, in h, and this is beyond the scope of this paper.)

For convenience, let P_T be max $(P_{T_\mathrm{w}}, P_{T_0}, P_{T_\mathrm{aw}})$ so that Eqs. (2.7) and (2.9) can be expressed as

$$\frac{P_h}{h} = \frac{P_T}{\theta_\mathrm{aw}} \left\{ \left(\frac{P_{T_\mathrm{w}}}{P_T}\right)^2 + (1-\Theta)^2 \left(\frac{P_{T_0}}{P_T}\right)^2 + \Theta^2 \left(\frac{P_{T_\mathrm{aw}}}{P_T}\right)^2 \right\}^{1/2} \bigg/ \left(\beta \frac{\partial \Theta}{\partial \beta}\right). \tag{2.12}$$

Θ and the uncertainties in temperature are taken to be known constants, so that the numerator of the RHS of Eq. (2.12) is invariant with time; the denominator, however, depends on h and t.

Following Yan and Owen (2002), an *amplification parameter*, Φ_h, is defined as

$$\Phi_h = \frac{P_h}{h} \bigg/ \frac{P_T}{\theta_\mathrm{aw}}. \tag{2.13}$$

From Eq. (2.12) it follows that

$$\Phi_h = \Gamma_\Theta \Gamma_\beta, \tag{2.14}$$

where

$$\Gamma_\Theta = \left\{ \left(\frac{P_{T_\mathrm{w}}}{P_T}\right)^2 + (1-\Theta)^2 \left(\frac{P_{T_0}}{P_T}\right)^2 + \Theta^2 \left(\frac{P_{T_\mathrm{aw}}}{P_T}\right)^2 \right\}^{1/2} \tag{2.15}$$

and

$$\Gamma_\beta = \left(\beta \frac{\partial \Theta}{\partial \beta}\right)^{-1}. \tag{2.16}$$

A special case arises when the uncertainties in the temperatures are equal to each other so that $P_{T_\mathrm{w}} = P_{T_0} = P_{T_\mathrm{aw}} = P_T$. Denoting this case by $*$, Eq. (2.15) simplifies to

$$\Gamma_\Theta^* = \left\{2(1 - \Theta + \Theta^2)\right\}^{1/2} \tag{2.17}$$

and

$$\Phi_h^* = \Gamma_\Theta^* \Gamma_\beta. \tag{2.18}$$

As $\Gamma_\Theta \leqslant \Gamma_\Theta^*$ for all Θ, Eq. (2.18) provides an upper bound for the uncertainty in h.

Eq. (2.14) can be regarded as the general expression for Φ_h. The actual value of Φ_h depends on the relationship between Θ and β, and the solutions for three cases are presented below.

3. Determination of amplification parameters

3.1. Step-change case

It follows from Eqs. (2.5) and (2.6) in 1 that

$$\frac{\partial \Theta}{\partial \beta} = \frac{\mathrm{d}f}{\mathrm{d}\beta} = 2[\beta(\Theta - 1) + \pi^{-1/2}]. \tag{3.1}$$

Hence, from Eqs. (2.14)–(2.16),

$$\Phi_h = \Gamma_\Theta / \{2\beta[\beta(\Theta - 1) + \pi^{-1/2}]\}, \tag{3.2}$$

which is the same as the result obtained by Yan and Owen (2002). It is interesting to note that, as β is uniquely related to Θ, Φ_h for the step-change case is independent of h.

For the special case where the uncertainties in temperature are equal, Eq. (2.17) leads to the result that

$$\Phi_h^* = \frac{[(1 - \Theta + \Theta^2)]^{1/2}}{\sqrt{2}\beta[\beta(\Theta - 1) + \pi^{-1/2}]}. \tag{3.3}$$

As $\Phi_h \leqslant \Phi_h^*$, Eq. (3.3) provides an upper bound for the amplification parameter.

The variation of Φ_h^* with Θ is shown in Fig. 2, and Yan and Owen found good agreement between Eq. (3.3) and computations made using a Monte Carlo method. A minimum value ($\Phi_{h,\mathrm{min}}^* \approx 4.4$) occurs at $\Theta \approx 0.52$, and $\Phi_h^* \lesssim 5$ for $0.3 < \Theta < 0.7$. In this range of Θ, 1% uncertainty in the temperature measurement would cause around 5% uncertainty in the computed value of h, a result most experimenters would regard as acceptable.

3.2. Exponential case

It follows from Eqs. (2.10) in 1 and (2.16) above that

$$\Gamma_\beta^{-1} = \beta \frac{\partial g}{\partial \beta}, \tag{3.4}$$

where it is assumed implicitly that there is no uncertainty in λ.

Hence, from the definition of $g(\beta, \lambda)$ given in Eq. (2.4),

$$\Gamma_\beta^{-1} = \frac{\beta\lambda^2}{(\beta^2 + \lambda^2)^2} \left\{ 2\beta[1 - f(\beta)] + (\beta^2 + \lambda^2)\frac{\mathrm{d}f}{\mathrm{d}\beta} - \mathrm{e}^{-\lambda^2}\left[2\beta + (1 - (\beta/\lambda)^2)\lambda\phi(\lambda)\right] \right\}, \tag{3.5}$$

where $\mathrm{d}f/\mathrm{d}\beta$ is given by Eq. (3.1). Eq. (3.5) can be used with Eqs. (2.15) and (2.17) to calculate Φ_h and Φ_h^* according to Eqs. (2.14) and (2.18).

For the special case of equal uncertainties in the temperatures, the effect of λ on the variation of Φ_h^* with Θ is shown in Fig. 3. For $\lambda > 2$, it can be seen that Φ_h^* for the exponential case tends to the value given by Eq. (3.3) for the step-change.

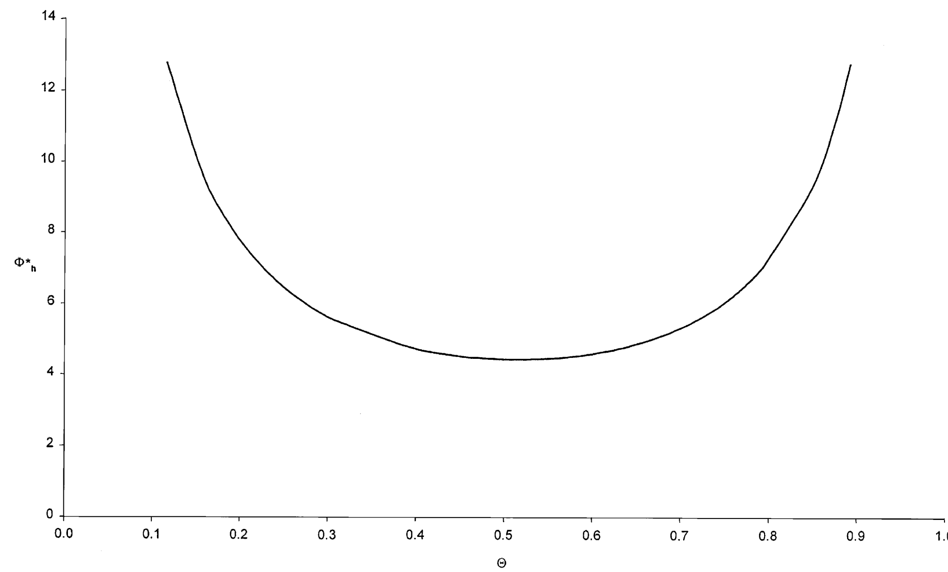

Fig. 2. Variation of Φ_h^* with Θ for step-change case.

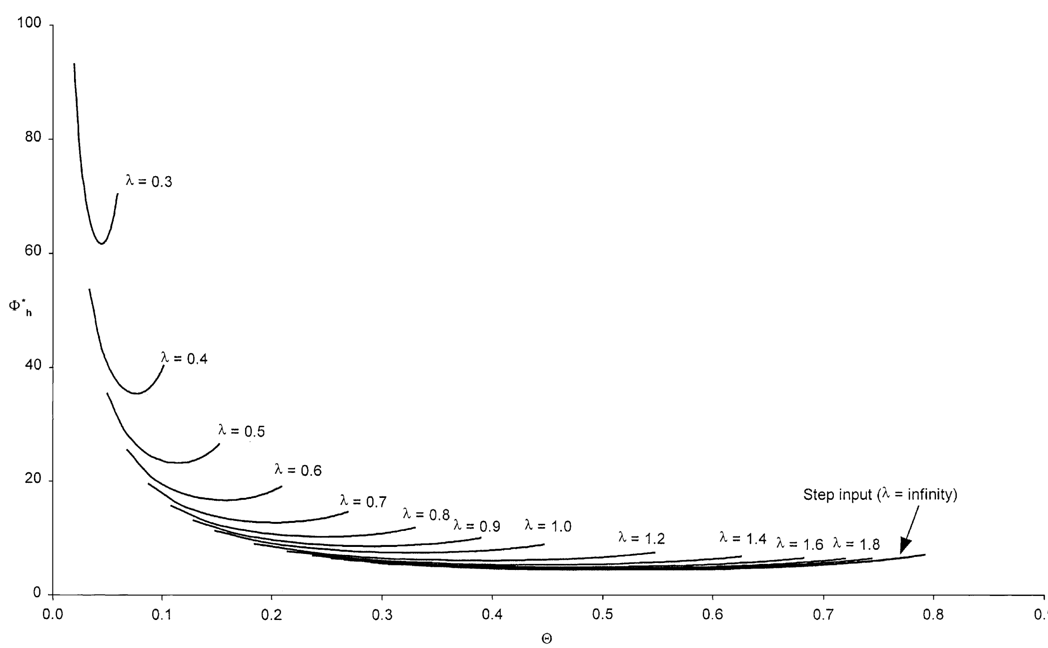

Fig. 3. Effect of λ on variation of Φ_h^* with Θ for exponential case.

In general, Φ_h^* for the exponential case is larger than that for the step change and, for small values of λ, it can be very large. For example, for $\lambda = 0.4$, $\Phi_h^* > 35$: 1% uncertainty in temperature results in more than 35% uncertainty in h. It is therefore desirable to make λ as large as practicable so as to reduce the uncertainty in h. For any value of λ, there is an optimum value of Θ that will minimise Φ_h^*, and this optimum value increases as λ increases: for $\lambda > 2$, $\Theta_{opt} \approx 0.52$. As τ is dependent on the experimental apparatus, the experimenter needs to choose both λ and Θ so as to achieve acceptably small values of Φ_h^* for the duration of the experiment.

Fig. 4 shows the effect of β_τ on the variation of Φ_h^* with Θ according to the equations derived above. In addition, computations were carried out using a Monte Carlo method (as described by Yan and Owen, 2002) in which random noise was added to T_w, T_o and T_{aw}, and the associated values of h were computed using Eq. (2.1) (with $m = 1$ and $c_1 = 1$). A total of 10,000 values was used to calculate the average value of h and its uncer-

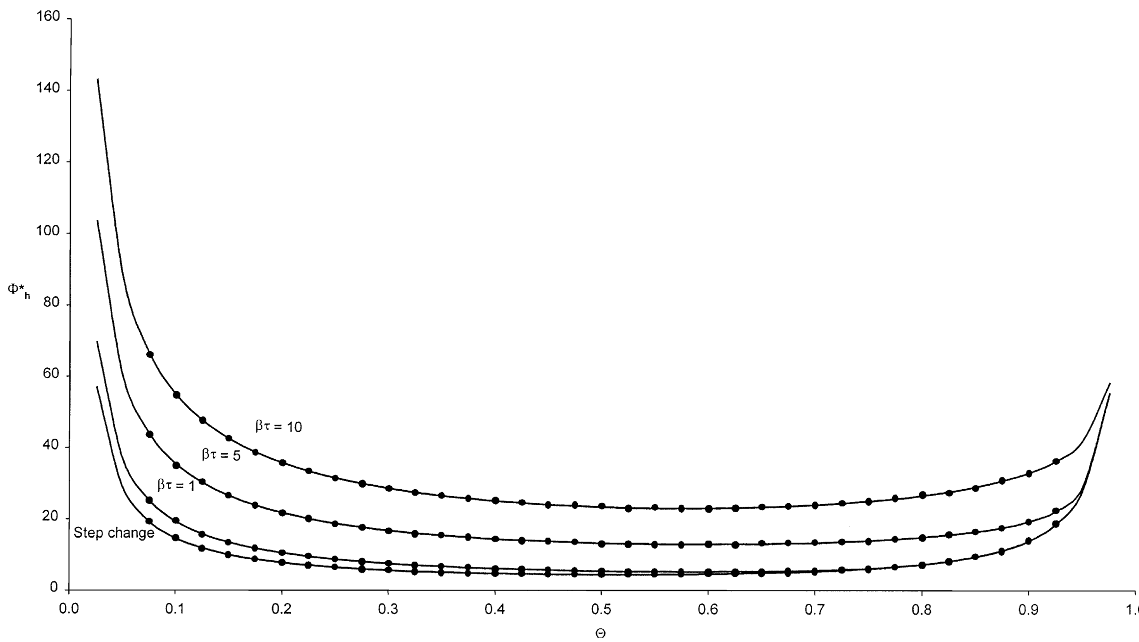

Fig. 4. Effect of β_τ on variation of Φ_h^* with Θ for exponential case (symbols refer to computed values).

tainty, P_h, for a wide range of values of Θ and β_τ. For the computations, $T_{aw} - T_0 = 40$ °C, $P_T = 0.2$ °C and $h = 100$ W/m^2 K, which are typical of the values found in experiments with TLC. The excellent agreement between the computations and the theoretical values of Φ_h^* gives confidence in the uncertainty analysis used here.

As pointed out in Section 2, Φ_h^* provides an upper bound for the uncertainty in h even when the uncertainties in the temperatures are unequal. Fig. 4 is therefore important in showing how the uncertainties in h behave with Θ and β_τ. It can be seen that the step-change case (where $\beta_\tau = 0$) provides the lower bound for Φ_h^*, and this limit is approached for $\beta_\tau < 1$.

The minimum value of Φ_h^* occurs for $0.5 < \Theta < 0.6$, depending on β_t. This is the range of Θ in which experiments with TLC should be conducted if the uncertainties in h are to be kept to a minimum. $\Phi_{h,\min}^*$ increases as β_τ increases: for $\beta_\tau = 1$, $\Phi_{h,\min}^* \approx 5$; for $\beta_\tau = 10$, $\Phi_{h,\min}^* \approx 23$. It therefore makes sense to design an experimental rig that makes τ as small as practicable.

3.3. Exponential series

Using Eqs. (2.1) and (2.16),

$$\Gamma_\beta^{-1} = \sum_{j=1}^{m} c_j \beta \frac{\partial}{\partial \beta}\left[g(\beta, \lambda_j)\right] \tag{3.6}$$

and from Eq. (2.14),

$$\Gamma_h^{-1} = \sum_{j=1}^{m} c_j \Phi_{h,j}^{-1}, \tag{3.7}$$

where

$$\Phi_{h,j}^{-1} = \Gamma_\Theta^{-1} \beta \frac{\partial}{\partial \beta}\left[g(\beta, \lambda_j)\right]. \tag{3.8}$$

That is, Φ_h^{-1} is the sum of the weighted $\Phi_{h,j}^{-1}$ components. When $m = 1$, $c_j = 1$ and Φ_h is the same as that given in Section 3.2.

For the special case where the temperature uncertainties are equal,

$$\Phi_h^{*^{-1}} = \sum_{j=1}^{m} c_j \Phi_{h,j}^{*^{-1}} \tag{3.9}$$

and Fig. 4 can be used to show the variation of $\Phi_{h,j}^*$ with Θ.

4. Conclusions

Using solutions of Fourier's equation for a semi-infinite wall, analytical expressions have been derived for the amplification factor, Φ_h, which relates the uncertainty in h to the uncertainties in the measured temperatures. Results have been obtained for the cases where the change of air temperature with time can be represented by: (i) a step change; (ii) a simple exponential; (iii) a series of exponentials. Cases (ii) and (iii) are referred to here as "slow transients". Excellent agreement was achieved between the analytical solutions for Φ_h and values computed using a Monte Carlo method for the special case where the uncertainties in the measured temperatures are equal to each other. This special case provides an upper bound for Φ_h when the uncertainties in the temperatures are unequal.

The results show that Φ_h will be a minimum for all three cases when the nondimensional temperature, Θ, is between 0.5 and 0.6. For the case of an exponential rise in air temperature, $\Phi_{h,\min}$ increases as β_τ increases, where β_τ is a nondimensional heat transfer coefficient. These results should provide useful guidance for experimenters in the selection of design parameters for a rig and for the calculation and minimisation of the uncertainty in h.

Acknowledgements

The authors wish to thank the Engineering and Physical Sciences Research Council (EPSRC) and Alstom Power Ltd for funding the research presented in this two-part paper.

References

Coleman, H.W., Steele, W.G., 1999. Experimentation and uncertainty analysis for engineers. Wiley, New York.

Newton, P.J., Yan, Y., Stevens, N.E., Evatt, S.T., Lock, G.D., Owen, J.M., 2003. Transient heat transfer measurements using thermochromic liquid crystal. Part 1: An improved technique, Int. J. Heat Fluid Flow 24, 14–22.

Yan, Y., Owen, J.M., 2002. Uncertainties in transient heat transfer measurements with liquid crystal. Int. J. Heat Fluid Flow 23, 29–35.